First difference principle applied to twin-width and merge-width of binary structures

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Abstract

In this article, we study twin-width and merge-width of binary structures under the light of first difference principle.

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1 Introduction

In this article, we continue our work of contextualisation of many graphs or binary structures widths in the framework of first difference principle. This work started in Lyaudet (2020). Before that, following Cantor (1895), Hausdorff (1907) et Sierpiński (1932), we started to study the first difference principle in Lyaudet (2018), Lyaudet (2019). We show that *twin-width* (introduced in Bonnet et al. (2020)) and *merge-width* (introduced in Dreier and Toruńczyk (2025)) fit very naturally in the framework of first difference principle.

2 Common definitions

For twin-width as well as for merge-width, we need the first difference principle extended with the special constant adjacency type nyf (*not yet fixed*). (There is also a special constant adjacency type alf (*already fixed*). We published both for the first time in Lyaudet (2020), but the idea of these two special adjacency types dates from the end of 2019, and alf is implicit in the definition of tree-questionable-width in Lyaudet (2019).)

Consider a binary signature $\mathcal{S}$ of unary and binary relations and functions. Given a set S, an $(\mathcal{S}, S, k, l)$-*mapping-run* is an (ordinal-indexed) sequence $(S_i)_{i \in l}$ of length l of $\mathcal{S}$-structures of cardinality at most k, k being the lowest such cardinal, together with a sequence of mappings f_i from S to the domains of S_i structures. Each S_i is an

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$\mathcal{S}$*-structure-item* (of the mapping-run/the $(S_i)_{i\in l}$ sequence). Each element/vertex x of S is thus associated to a sequence $(x_i = f_i(x))_{i\in l}$; this sequence is an *"element"* of the mapping-run/the $(S_i)_{i\in l}$ sequence; it corresponds to an element/vertex of S; each $x_i = f_i(x)$ is an *element/vertex-item.*

Definition 2.1 (nyf-extended question)**.** *Given two elements X, Y of a mapping-run, we say that (q, x_q, y_q, S_q) is the* question *of X, Y, if q is the smallest ordinal such that $x_q \neq y_q$ and the adjacency type between x_q and y_q is not* nyf.

3 Twin-width

Definition 3.1 (Twin-width)**.** *A* twin-decomposition *of an $\mathcal{S}$-structure S is a sequence of $\mathcal{S}$-structures indexed by an ordinal $(S_i)_{i\in l+1}$ such that $S_0 = S$ and S_l is a singleton. We always go from S_i to S_{i+1} by a unique merging/contraction of two vertices of S_i into one. Given two vertices $x, y \in S_i$, we say the edge between these vertices is* red *if the adjacencies in the starting binary structure are of at least two distinct types between the vertices corresponding to $f_i^{-1}(x)$ and those corresponding to $f_i^{-1}(y)$. The* maximum red degree *of an $\mathcal{S}$-structure S_i is equal to the supremum of the degrees of its vertices when we only keep the red edges. The width of a twin-decomposition $(S_i)_{i\in l+1}$ is equal to the supremum of the maximum red degrees of the S_i. The* twin-width *of an $\mathcal{S}$-structure S, denoted by* $\mathrm{tww}(S)$*, is the minimum of the widths of all its twin-decompositions.*

If we do not focus on the size of the structure-items, but look instead at the degree of the nyf adjacencies, the equivalence with nyf-extended first difference principle is quite clear. We only need to reverse the decomposition to consider separations/splitting.

Definition 3.2 (Twin-questionable-width)**.** *A* twin-questionable-decomposition *of an $\mathcal{S}$-structure S is an $(\mathcal{S}, S, k, l+1)$-mapping-run (a sequence of $\mathcal{S}$-structures indexed by an ordinal $(S_i)_{i\in l+1}$) such that S_0 is a singleton and $S_l = S$. We always go from S_i to S_{i+1} by a unique separation/splitting of a vertex of S_i in two vertices. The* maximum nyf degree *of an $\mathcal{S}$-structure S_i is equal to the supremum of the degrees of its vertices when we only keep the* nyf *edges. The width of a twin-questionable-decomposition $(S_i)_{i\in l+1}$ is equal to the supremum of the maximum* nyf *degrees of the S_i. The* twin-questionable-width *of an $\mathcal{S}$-structure S, denoted by* $\mathrm{twqw}(S)$*, is the minimum of the widths of all its twin-questionable-decompositions.*

Theorem 3.3. *The twin-width is equal to the twin-questionable-width.*

4 Merge-width

Definition 4.1 ((Weak) merge-width)**.** *Fix a vertex set and an $\mathcal{S}$-structure S. A* construction sequence *is a sequence of steps, maintaining a partition $\mathcal{P}$ of S and a partition of $\binom{S}{2}$ into two sets: resolved "edges" $\mathcal{R}$ with a given adjacency type (it can correspond to edges and non-edges of a graph), and unresolved pairs $\mathcal{U}$. Initially, $\mathcal{P}$*

partitions S into singletons, and every pair in $\binom{S}{2}$ is unresolved. In each step, one of two operations is performed:

- *merge two parts $X, Y \in \mathcal{P}$, replacing the two parts by their union,*
- *resolve a pair of parts $X, Y \in \mathcal{P}$ by a given adjacency type (possibly $X = Y$ except in the weak case), declaring all the unresolved pairs $\{a, b\} \in \mathcal{U}$ with $a \in X$ et $b \in Y$ to be of the chosen adjacency type, that is, moving them from $\mathcal{U}$ to $\mathcal{R}$.*

In the end, we require that $\mathcal{P}$ has one part, and that every pair from $\binom{S}{2}$ is resolved with a valid adjacency type. We thus say this is a construction sequence *of S. The* radius-r width *of a construction sequence is the least number k such that at every step in the sequence, the following holds: For every vertex $v \in S$, at most k parts of the current partition $\mathcal{P}$ can be reached from v by a path of length r in the graph of the currently resolved adjacencies. The* radius-r merge-width *of S, denoted by* $\mathrm{mw}_r(S)$ *or* $\mathrm{wmw}_r(S)$ *in the weak case, is the least radius-r width of a construction sequence of S. Finally, a graph class $\mathcal{C}$ has* bounded merge-width*, resp.* bounded weak-merge-width*, if $\mathrm{mw}_r(\mathcal{C}) < \infty$, resp. $\mathrm{wmw}_r(\mathcal{C}) < \infty$, for all $r \in \mathbb{N}$, where $\mathrm{mw}_r(\mathcal{C}) = \sup_{S \in \mathcal{C}} \mathrm{mw}_r(S)$, resp. $\mathrm{wmw}_r(\mathcal{C}) = \sup_{S \in \mathcal{C}} \mathrm{wmw}_r(S)$.*

This is a naming problem:

- $\mathcal{R}$ for resolved corresponds to $\mathcal{A}$ for alf.
- $\mathcal{U}$ for unresolved corresponds to $\mathcal{N}$ for nyf. Unresolved is just another word for nyf.

Here again, if we do not focus on the size of the structure-items, but look instead the condition on the size of the non-nyf balls, the equivalence with nyf-extended first difference principle for the weak-merge-width is very clear. For the merge-width, we need an extension that is bordeline first difference principle:

Definition 4.2 (nyf-pure question)**.** *Given two elements X, Y of a mapping-run, we say that (q, x_q, y_q, S_q) is the* question *of X, Y, if q is the smallest ordinal such that the adjacency type between x_q and y_q is not* nyf. *If $x_q = y_q$, we are looking at the adjacency type on a loop; it can only work for symmetric adjacency types; in particular, the merge-width of directed graphs is partially downgraded to weak-merge-width; the merge-width of tournaments is totally downgraded to weak-merge-width.*

All the other definitions of question can be simulated by this one with the convention that the loops have only the nyf adjacency type. It makes us lose the possibility to consider that the loops on the S_i are only here to fix the loops of S. This is not a problem in the case of merge-width, because we start with all vertices separated; they can thus receive their loops by questionability with S_0, and after that the loops of the S_i only fix adjacencies between vertices.

Definition 4.3 ((Weak) merge-questionable-width)**.** *Let S be a set of vertices and an $\mathcal{S}$-structure. A* merge-questionable-decomposition *is an $(\mathcal{S}, S, p, l)$-mapping-run, a* *sequence of $\mathcal{S}$-structures indexed by an ordinal $(S_i)_{i\in l+1}$ such that $S_0 = S$ and S_l is a singleton. We always go from S_i to S_{i+1} by a unique merging/contraction of two vertices of S_i into one.*

The radius-r width *of a merge-questionable-decomposition of S is the smallest number k such that at each step i, it is true that: For each vertex $v \in S$, at most k vertices of S_i can be reached from v by a path of length at most r in the graph of adjacencies that were fixed until now: by the* nyf*-extended first difference principle in the weak-case, by the* nyf*-pure questionability otherwise. The* radius-r merge-questionable-width *of S, denoted by* $\mathrm{mqw}_r(S)$ *or* $\mathrm{wmqw}_r(S)$ *in the weak case, is the smalled* radius-r width *among the merge-questionable-decompositions of S. Finally, a graph class $\mathcal{C}$ has* bounded merge-questionable-width*, resp.* bounded weak-merge-questionable-width*, if* $\mathrm{mqw}_r(\mathcal{C}) < \infty$*, resp.* $\mathrm{wmqw}_r(\mathcal{C}) < \infty$*, for all $r \in \mathbb{N}$, where* $\mathrm{mqw}_r(\mathcal{C}) = \sup_{S\in\mathcal{C}} \mathrm{mqw}_r(S)$*, resp.* $\mathrm{wmw}_r(\mathcal{C}) = \sup_{S\in\mathcal{C}} \mathrm{wmqw}_r(S)$.

Theorem 4.4. *The radius-r merge-questionable-width is equal to the radius-r merge-width. Hence, the merge-width is bounded if and only if the merge-questionable-width is bounded.*

Theorem 4.5. *The radius-r weak-merge-questionable-width is equal to the radius-r weak-merge-width. Hence, the weak-merge-width is bounded if and only if the weak-merge-questionable-width is bounded.*

In the first version of this article, we conjectured that the graphs of bounded degree do not have bounded weak-merge-width, although they have bounded merge-width. Szymon Toruńczyk read this first version, and was kind enough to make us the remark that bounded weak-merge-width should be equivalent to bounded twin-width. We detail the proof skeleton given by Szymon Toruńczyk in the following section.

5 Combination of the two widths

We continue by setting naming conventions that are more explicit and less ambiguous. We have already set the terms of question for the classical first difference, of nyf-extended question for a first extension of first difference principle, and of nyf-pure question for a second extension of first difference principle that doesn't rely anymore on a mandatory difference. We will say that a question is simple when it is according to classical first difference principle. It seems appropriate to us to use the adjectives "*questionnable*" (with two "n" like the French word), "questionneble", "questionpable", in parallel of these 3 types of questions; "questionneble" and "questionpable" are neologisms to insert the "e" of "extended" and the "p" of "pure". One can also associate the shorten terms "q", "qs", "qe", "qp". The inclusive terms will be "question" and "questionable" written with only one "n" like the English word. To summarize, we propose the following conventions:

question	questionable	q
simple question	questionnable	qs
nyf-extended question	questionneble	qe
nyf-pure question	questionpable	qp

It is also important to explicit a common trait of all adjacency fix by a question: the counterimage vertices of the two image vertices must have homogeneous adjacencies if we exclude already fixed adjacencies. This principle will be called HIEAA, for "Homogenous If Excluding alf-Adjacencies". HIEAA reminds of a cowboy scream; the first difference principle can even give orders to horses ;) XD.

Lemma 5.1. *When we consider partitions of the set of vertices more and more coarse (a set of singletons being the finest partition), no questionable decomposition can work if at some point two parts are not HIEAA.*

This lemma applies notably to the contraction/construction sequences of the twin-width and the merge-width.

In Dreier and Toruńczyk (2025), Example 1.3, that shows that bounded twin-width implies bounded merge-width, shows in reality that the weak-merge-width is bounded. This proof rely on the idea to fix, before contracting two parts A and B, all the adjacencies between a part C and the parts A and B, when C is not HIEAA with A and B taken together. Fixing also the adjacency between the parts A and B is not a problem.

Hence :

Lemma 5.2. *If twin-width is bounded, then weak-merge-width is bounded. And, in particular,* $\mathrm{tww}(S) \leq k$ *implies* $\mathrm{wmw}_2(S) \leq 2 + k + k^2$.

Conversely, if $\mathrm{wmw}_2(S) \leq k$, then by Lemma 5.1 $\mathrm{tww}(S) \leq k - 1$.

Lemma 5.3. *If the weak-merge-width is bounded, then the twin-width is bounded.*

Theorem 5.4 (Toruńczyk 2025)**.** *Twin-width is bounded if and only if weak-merge-width is bounded.*

Inspired by this equivalence, we propose other widths that are more or less similar to twin-width.

Definition 5.5 (Non-nyf questionable widths)**.** *A* non-nyf questionable decomposition *of an $\mathcal{S}$-structure S is an $(\mathcal{S}, S, k, l+1)$-mapping-run (a sequence of $\mathcal{S}$-structures indexed by an ordinal $(S_i)_{i \in l+1}$) such that $S_0 = S$ and S_l is a singleton. We always go from S_i to S_{i+1} by a unique merging/contraction of two vertices of S_i into one. The* non-nyf maximum degree *of an $\mathcal{S}$-structure S_i is equal to the supremum of the degrees of its vertices when we only keep the non-*nyf *edges. The* non-nyf cardinal *of an $\mathcal{S}$-structure S_i is equal to the cardinality of the set of non-*nyf *edges.*

If the decomposition uses the nyf*-extended first difference principle to fix all the adjacencies, we talk about questionneble decomposition. If the decomposition uses the* nyf*-pure questionability to fix all the adjacencies, we talk about questionpable decomposition.*

The width of a non-nyf *maximum degree questionneble, resp. questionpable, decomposition $(S_i)_{i \in l+1}$ is equal to the supremum of the maximum non-*nyf *degrees of the S_i. The* non-nyf maximum degree questionneble, resp. questionpable, width *of an $\mathcal{S}$-structure S, denoted by* $\neg\mathrm{nyf}\Delta\mathrm{qew}(S)$, *resp.* $\neg\mathrm{nyf}\Delta\mathrm{qpw}(S)$, *is the minimum of the widths of all its non-*nyf *maximum degree questionneble, resp. questionpable, decompositions.*

The width of a non-nyf *cardinal questionneble, resp. questionpable, decomposition* $(S_i)_{i\in l+1}$ *is equal to the supremum of the non-*nyf *cardinals of the* S_i*. The* non-nyf cardinal questionneble, resp. questionpable, width *of an* $\mathcal{S}$*-structure* S*, denoted by* $\neg\mathrm{nyf}\sharp\mathrm{qew}(S)$*, resp.* $\neg\mathrm{nyf}\sharp\mathrm{qpw}(S)$*, is the minimum of the widths of all its non-*nyf *cardinal questionneble, resp. questionpable, decompositions.*

Obviously:

Lemma 5.6. $\neg\mathrm{nyf}\Delta\mathrm{qew}(S) \leq k$ *implies* $\neg\mathrm{nyf}\Delta\mathrm{qpw}(S) \leq k$*.* $\neg\mathrm{nyf}\sharp\mathrm{qew}(S) \leq k$ *implies* $\neg\mathrm{nyf}\sharp\mathrm{qpw}(S) \leq k$*.*

Lemma 5.7. $\neg\mathrm{nyf}\sharp\mathrm{qew}(S) \leq k$ *implies* $\neg\mathrm{nyf}\Delta\mathrm{qew}(S) \leq k$*.* $\neg\mathrm{nyf}\sharp\mathrm{qpw}(S) \leq k$ *implies* $\neg\mathrm{nyf}\Delta\mathrm{qpw}(S) \leq k$*.*

And by the HIEAA principle and the fact that one can always add the adjacency between the two contracted parts just before:

Lemma 5.8. $\neg\mathrm{nyf}\Delta\mathrm{qpw}(S) \leq k$ *implies* $\neg\mathrm{nyf}\Delta\mathrm{qew}(S) \leq k+1$*.* $\neg\mathrm{nyf}\sharp\mathrm{qpw}(S) \leq k$ *implies* $\neg\mathrm{nyf}\sharp\mathrm{qew}(S) \leq k+1$*.*

Again by the construction in Example 1.3 in Dreier and Toruńczyk (2025), we have:

Lemma 5.9. $\mathrm{tww}(S) \leq k$ *implies* $\neg\mathrm{nyf}\Delta\mathrm{qew}(S) \leq k+1$*.* $\mathrm{tww}(S) \leq k$ *implies* $\neg\mathrm{nyf}\sharp\mathrm{qew}(S) \leq 2 \times k + 1$*.*

But it is easy to see that:

Lemma 5.10. *The bounded degree graphs have a bounded non-*nyf *maximum degree questionneble and questionpable width.*

We think that the graphs of bounded degree do not have bounded $\neg\mathrm{nyf}\sharp\mathrm{qew}$. Whilst it seems plausible that the merge-width is not bounded for some classes of graphs of bounded $\neg\mathrm{nyf}\Delta\mathrm{qew}$.

Finally, we add a monotony constraint to the $\neg\mathrm{nyf}\Delta\mathrm{qew}$ and $\neg\mathrm{nyf}\sharp\mathrm{qew}$ widths by requiring that no non-nyf edge can become nyf in a following structure-item. It defines the $\mathrm{m}\neg\mathrm{nyf}\Delta\mathrm{qew}$ and $\mathrm{m}\neg\mathrm{nyf}\sharp\mathrm{qew}$ widths.

$\mathrm{m}\neg\mathrm{nyf}\sharp\mathrm{qew}$ becomes relatively weak because it only handles a finite global number of “non-modular” edges. However, $\mathrm{m}\neg\mathrm{nyf}\Delta\mathrm{qew}$ gives us a fourth width equivalent to twin-width.

Lemma 5.11. $\mathrm{m}\neg\mathrm{nyf}\Delta\mathrm{qew}(S) \leq k$ *implies* $\mathrm{m}\neg\mathrm{nyf}\Delta\mathrm{qpw}(S) \leq k$*.* $\mathrm{m}\neg\mathrm{nyf}\sharp\mathrm{qew}(S) \leq k$ *implies* $\mathrm{m}\neg\mathrm{nyf}\sharp\mathrm{qpw}(S) \leq k$*.*

Lemma 5.12. $\mathrm{m}\neg\mathrm{nyf}\sharp\mathrm{qew}(S) \leq k$ *implies* $\mathrm{m}\neg\mathrm{nyf}\Delta\mathrm{qew}(S) \leq k$*.* $\mathrm{m}\neg\mathrm{nyf}\sharp\mathrm{qpw}(S) \leq k$ *implies* $\mathrm{m}\neg\mathrm{nyf}\Delta\mathrm{qpw}(S) \leq k$*.*

Lemma 5.13. $\mathrm{m}\neg\mathrm{nyf}\Delta\mathrm{qpw}(S) \leq k$ *implies* $\mathrm{m}\neg\mathrm{nyf}\Delta\mathrm{qew}(S) \leq k+1$*.* $\mathrm{m}\neg\mathrm{nyf}\sharp\mathrm{qpw}(S) \leq k$ *implies* $\mathrm{m}\neg\mathrm{nyf}\sharp\mathrm{qew}(S) \leq k+1$*.*

Lemma 5.14. $\mathrm{tww}(S) \leq k$ *implies* $\mathrm{m}\neg\mathrm{nyf}\Delta\mathrm{qew}(S) \leq k+1$*.*

By monotony, it is useless to fix adjacencies way before contracting incident vertices (since it only increases the maximum degree); hence we can consider non-nyf edges as red edges.

Lemma 5.15. $\mathrm{m}\neg\mathrm{nyf}\Delta\mathrm{qew}(S) \leq k$ *implies* $\mathrm{tww}(S) \leq k$.

It seems that the studied widths go in order from the weakest decomposition power to the strongest decomposition power from:

- $\mathrm{m}\neg\mathrm{nyf}\sharp\mathrm{qew} \approx \mathrm{m}\neg\mathrm{nyf}\sharp\mathrm{qpw}$,
- $\mathrm{tww} = \mathrm{twqw} \approx \mathrm{wmw} = \mathrm{wmqw} \approx \mathrm{m}\neg\mathrm{nyf}\Delta\mathrm{qew} \approx \mathrm{m}\neg\mathrm{nyf}\Delta\mathrm{qpw}$,
- $\neg\mathrm{nyf}\sharp\mathrm{qew} \approx \neg\mathrm{nyf}\sharp\mathrm{qpw}$,
- $\mathrm{mw} = \mathrm{mqw}$,
- to $\neg\mathrm{nyf}\Delta\mathrm{qew} \approx \neg\mathrm{nyf}\Delta\mathrm{qpw}$.

6 Conclusion

The graphs of bounded degree have also a bounded bijective balanced tree-questionable-width (see Lyaudet (2022)); but this is not the case with the twin-width. It would be interesting to know how the twin-width, the merge-width, and the non-nyf maximum degree questionable width compare to the distinct variants of tree-questionable-width (see Lyaudet (2025b)).

These translations of widths in the framework of first difference principle or nyf-pure questionability may seem inconsequential. But these translations can be expressed themselves in some logics, and they operate only on sequences of elements, hence on structures of "paths" and not more complicated graphs. It is possible that more general results on the first difference principle or the nyf-pure questionability enable to classify the complexity of computing these various widths, between those that are in P or NC, or those that are NP-hard with or without approximation algorithms, etc.

Thanks God! Thanks Father! Thanks Jesus! Thanks Holy-Spirit!

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